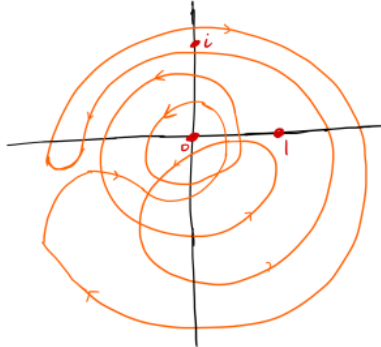Math 421 HW7 Additional Problems

1. Let C be the curve depicted below.

   (a) What are the winding numbers of C about $z = 0$, $z = 1$, and $z = i$?

   (b) Compute $\displaystyle\int_C \frac{\cos(i\pi z)}{z - z_0}\, dz$, where $z_0 = 0$, 1, and i.

2. Let f be analytic on a simply connected domain D containing the positively oriented circle of radius 3 centered at $1 + i$. If the maximum value of $|f(z)|$ on the circle is 7, show that $|f'''(1 + i)| \leq \frac{14}{9}$.

3. Let f be entire and suppose $f(z) = c$ for all z on the circle $|z| = R$, where c is a complex number and $R > 0$. Show that $f(z) = c$ for all z in the disk $|z| \leq R$. Be sure to explain your reasoning thoroughly.