Practice Problems

1.) Consider the bases $\mathcal{E}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{Z}^3$

Let $Q : \mathbb{Z}^3 \times \mathbb{Z}^3 \to \mathbb{Z}$ be the symmetric bilinear form given by $Q\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \right) = x_1 x_2 - y_1 y_2 + z_1 z_2$

Write down matrix representations of Q in each basis

2.) Show that if Q is positive or negative definite, then Q is nondegenerate.

3.) Show that if Q is positive/negative definite, then the diagonal entries of $Q_{\mathcal{B}}$ are positive/negative for any basis $\mathcal{B}$.

4.) Show that if Q is positive/negative definite, then the eigenvalues of $Q_{\mathcal{B}}$ are positive/negative for any basis $\mathcal{B}$.

(Note: the converse is also true)