

# Practice Problems

1.) Let  $Q = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ .

We know  $\nexists$  lattice embedding  $(\mathbb{Z}^2, Q) \rightarrow (\mathbb{Z}^2, -I)$ .

Does  $\exists$  a lattice embedding  $\varphi: (\mathbb{Z}^2, Q) \rightarrow (\mathbb{Z}^3, -I)$ ?

If so, sketch  $\text{Im} \varphi$ .

2.) Let  $Q = \begin{bmatrix} -n & 1 \\ 1 & -2 \end{bmatrix}$ . For what values of  $n \geq 2$  does

there exist a lattice embedding  $\varphi: (\mathbb{Z}^2, Q) \rightarrow (\mathbb{Z}^2, -I)$ ?

For at least one of lattice embeddings, sketch the sublattice  $\text{Im} \varphi \subset \mathbb{Z}^2$ .

\*3.) Let  $Q = \begin{bmatrix} -n & 1 & & 0 \\ 1 & -2 & & \\ & & \ddots & \\ 0 & & & 1 & -2 \end{bmatrix}$  be an  $(m+1) \times (m+1)$  matrix.

For which values of  $n \geq 2$  does there exist a lattice embedding  $\varphi: (\mathbb{Z}^{m+1}, Q) \rightarrow (\mathbb{Z}^{m+1}, -I)$