Practice Problems

1.) Let $Q = \begin{bmatrix} -n & 1 \\ 1 & -2 \end{bmatrix}$. Use linear algebra to decide for which values of $n \geq 2$ there exists a cubiquitous lattice embedding $\varphi: (\mathbb{Z}^2, Q) \to (\mathbb{Z}, -I)$. Sketch an example that is not cubiquitous.

*2.) Let $S = \{v_1, \dots, v_n\} \subset \mathbb{Z}^n$ be a standard subset with Wu element $W = \sum_{i=1}^{n} k_i e_i$.

Let $R_o = \{ i \mid k_i \text{ is odd} \}$, $R_e = \{ i \mid k_i \neq 0 \text{ is even} \}$, and $O = \{ i \mid k_i = 0 \}$

Show that if $\sum_{i \in R_o} \frac{k_i^2 - 1}{4} + \sum_{j \in R_e} \frac{k_j^2 - 4}{4} > |O|$

or $\sum_{i \in R_o} \frac{k_i^2 - 1}{4} + \sum_{j \in R_e} \frac{k_j^2 - 4}{4} = |O|$ and $|R_e| \geq 2$,

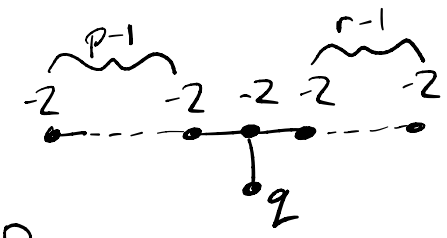
then the lattice generated by S is not cubiquitous

3.) Let Q be the incidence matrix for 

For which values of $m \geq 1$ and $n \geq 2$ does there exist a cubiquitous lattice embedding $\varphi: (\mathbb{Z}^{m+1}, Q) \to (\mathbb{Z}^{m+1}, -I)$?

(Hint: Use problem 2)

4.) Let Q be the incidence matrix for



where $p, r > 1$, $q < -1$, and $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} > 0$

Then $\exists$ a lattice embedding $\varphi: (\mathbb{Z}^{p+r}, Q) \to (\mathbb{Z}^{p+r}, -I)$ if and only if $-q = pl^2 + r(l+1)^2$ for some $l \in \mathbb{Z}$.

a) This is shown in Greene-Jabuka.
Read p.11 paragraph 3 up to Section 3.2 on p.12 in the Greene-Jabuka paper and write up the proof in your own words.

*b) Show that such a lattice embedding is cubiquitous if and only if $l = 0$ or $l = -1$.