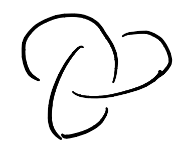
# Practice Problems

1.) Show that the following knots are slice (by hand or using KLO):

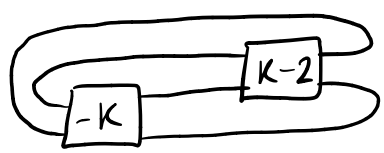
$8_8$, $8_9$, $8_{20}$ (see pictures on knot info)

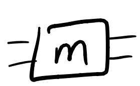
Feel free to also try:

$9_{27}, 9_{41}, 9_{46}, 10_3, 10_{22}, 10_{35}, 10_{42}, 10_{48}, 10_{75}, 10_{87}, 10_{99}, 10_{123}, 10_{129}, 10_{137}, 10_{140}, 10_{153}, 10_{155}$

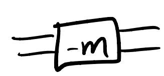
2.) For a knot $K$, let $-K$ denote its mirror image. Show that $K \# -K$ is slice for $K =$ 

(Note: this is true for all knots)

3.) Let $L_K =$ 


where 

is 

and 

is 


a) Show that $L_K$ is $\chi$-slice $\forall\, K \geq 2$

(we did $L_2$, $L_3$, and $L_4$ in class)

b) How many components does $L_K$ have? Which $\chi$-slice surface does it bound?

Note: these are related to the lattice embeddings $(\mathbb{Z}^{m+1}, Q) \to (\mathbb{Z}^{m+1}, -I)$

where $Q = \begin{bmatrix} -n & 1 & & 0 \\ 1 & -2 & 1 & \\ & 1 & \ddots & 1 \\ 0 & & 1 & -2 \end{bmatrix}$ from last week

4.) For a Knot $K$,

$g_3(K) = \min\{\text{genus of } \Sigma \mid \Sigma \text{ is an orientable surface embedded in } S^3 \text{ with } \partial\Sigma = K\}$

$g_4(K) = \min\{\text{genus of } \Sigma \mid \Sigma \text{ is an orientable surface embedded in } B^4 \text{ with } \partial\Sigma = K\}$

a) Show that $g_4(K) \le g_3(K) \quad \forall K$.

b) Give an example of a Knot with $g_3(K) = g_4(K)$.

c) Give an example of a Knot with $g_3(K) = 1$ and $g_4(K) = 0$.

(Note: It is hard in general to determine $g_3$, $g_4$. This question is the basis of ongoing research in knot theory)