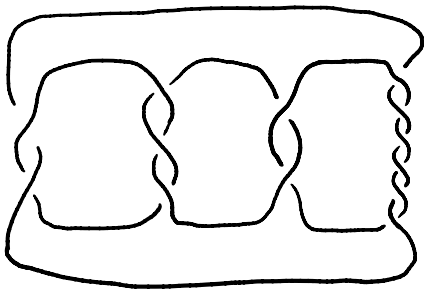
Practice Problems

1.) a.) What are the surfaces with $\chi = 1$ and 3 boundary components?

b.) Read Lemma 3.1 in "On χ-slice pretzel links."

c.) Use a) and c) to show that the following link is not χ-slice

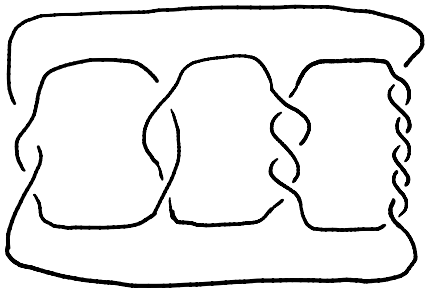
$P(2,-3,2,-6) =$



d.) Read about mutant knots/links.

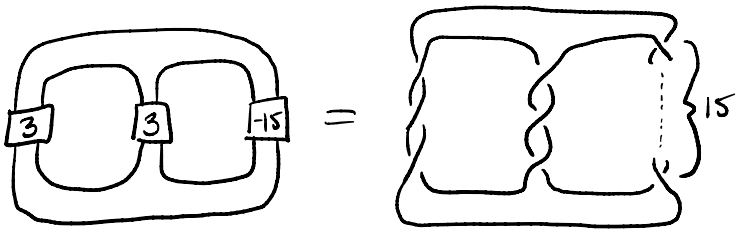
Show that

$P(2,2,-3,-6) =$

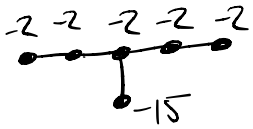


is a χ-slice mutant of $P(2,-3,2,-6)$.

Note: Many knot invariants are identical for mutants. For example, if K and K' are mutants, then $\sigma(K) = \sigma(K')$ and $\det K = \det K'$.

2.) Consider the knot $P(3,3,-15)$ = 

a) Calculate $\sigma(P(3,3,-15))$ and $\det(P(3,3,-15))$ by hand and use KLO to check. Do these obstruct sliceness?

b) If K is slice, $\exists$ lattice embedding $(\mathbb{Z}^n, Q) \to (\mathbb{Z}^n, -I)$ where Q is the incidence matrix of 

. Show that K is not slice by showing $\nexists$ such a lattice embedding.

3.) If V is a Seifert Matrix for a Knot K, then the Alexander polynomial of K is defined by $\Delta_K(t) = \det(V - tV^T)$. (Note that $\Delta_K(-1) = \det K$)

a) Compute $\Delta_K(t)$ for 

b) Read about the Fox-Milnor Theorem, which is another classical sliceness obstruction.

c) Can you show $P(3,3,-15)$ is not slice using this obstruction?

Note: $\Delta_K(t)$ is defined up to multiplication by $\pm t^n$, $n \in \mathbb{Z}$

e.g. $5 - 2t + 5t^2 \doteq 5t^3 - 2t^4 + 5t^5 \doteq 5t^{-2} - 2t^{-1} + 5$

It is often expressed as a symmetric Laurent polynomial. For example:

$$5 - 2t + 5t^2 \doteq 5t^{-1} - 2 + 5t$$

The Fox-Milnor theorem views $\Delta_K(t)$ as written in its symmetric form.