Final Exam Information

Visit Spire to find out the time and location of your final exam.

The final exam will focus on vector calculus. In particular, there will be an emphasis on computations and applications of line integrals, surface integrals, Fundamental Theorem of Line Integrals, Green's Theorem, Stokes' Theorem, the Divergence Theorem, the Generalized Stokes' Theorem, and differential forms.

To effectively work through these problems you also need to be comfortable with the earlier topics of parametrization, double/triple integrals, and vector fields. The relevant textbook sections are

$$2.4, 4.2 - 4.4, 5.1 - 5.5, 6.1 - 6.2, 7.1 - 7.6, 8.1 - 8.5$$

Some topics that we covered throughout the semester WILL NOT APPEAR on the final. These are the topics covered in lectures $9/4 - 9/20$, 10/4, 10/11, and 10/21. These lectures covered differentiability, critical points, extreme values, the second derivative test, optimization, the application of integrals to centers of mass and moments of inertia, and the notions of curvature of curves and curvature of surfaces (Gaussian and mean curvature).

In preparation for the final, I recommend looking over the solutions to the past exams and homework, reviewing your notes, and working through additional problems from the textbook. Below is a list of suggested problems. Of course, if you feel like you would like more practice, you can work through additional problems.

Section 4.2 # 1, 3, 7
Section 4.3 # 1, 3, 5, 7, 9, 21
Section 4.4 # 1, 3, 5, 7, 13, 15, 17, 21, 23, 27, 33
Section 5.5 # 1, 7, 9, 11, 13
Chapter 5 Review Exercises # 1, 3, 5, 7, 9, 15
Chapter 6 Review Exercises # 3, 5, 7, 15
Section 7.1 # 5, 7, 9, 11
Section 7.2 # 1, 3, 5, 6, 9, 11, 16, 17
Section 7.3 # 5, 7, 9
Section 7.4 # 1, 9
Section 7.5 # 1, 3, 9
Section 7.6 # 1, 3, 7, 13
Section 8.1 # 7, 9, 11
Section 8.2 # 7, 11, 13, 17
Section 8.3 # 1, 3, 7, 9, 13, 19
Section 8.4 # 5, 7, 9, 13, 24
Section 8.5 # 1, 3, 5, 11

Formulas: As with the previous exams, I will provide the following list of formulas on the exam so that you do not have to memorize them.

1. **Cylindrical Coordinates:** $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$$\iiint f(x,y,z)\,dx\,dy\,dz = \iiint f(r\cos\theta, r\sin\theta, z) r\,dz\,dr\,d\theta$$

2. **Spherical Coordinates:** $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$

$$\iiint f(x,y,z)\,dx\,dy\,dz = \iiint f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

3. The **arc length** of a parametrized curve $\mathbf{c} : [a,b] \to \mathbb{R}^n$ is $\displaystyle\int_a^b ||\mathbf{c}'(t)||\,dt$.

4. The **surface area** of a surface S parametrized by $\mathbf{r} : D \subset \mathbb{R}^2 \to \mathbb{R}^3$ is $\displaystyle\iint_D ||\mathbf{r}_u \times \mathbf{r}_v||\,du\,dv$

5. The **divergence** of a vector field $\mathbf{F}$ is div$\mathbf{F}$= $\nabla \cdot \mathbf{F}$ and the **curl** of $\mathbf{F}$ is curl$\mathbf{F}$= $\nabla \times \mathbf{F}$.

6. Let C be a curve parametrized by $\mathbf{c} : [a,b] \to \mathbb{R}^n$.
   The **line integral** of a function f over C is $\displaystyle\int_C f\,ds = \int_a^b f(\mathbf{c}(t))||\mathbf{c}'(t)||\,dt$
   The **line integral** of a vector field F over C is $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\,dt$.

7. Let S be a surface parametrized by $\mathbf{r} : D \subset \mathbb{R}^2 \to \mathbb{R}^3$.
   The **surface integral** of a function f over S is $\displaystyle\iint_S f\,dS = \iint_D f(\mathbf{r}(u,v))||\mathbf{r}_u \times \mathbf{r}_v||\,du\,dv$.
   The **surface integral** of a vector field $\mathbf{F}$ over S is $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\,du\,dv$.

8. **The Fundamental Theorem of Line Integrals:** Let C be parametrized by $\mathbf{c} : [a,b] \to \mathbb{R}^n$.

$$\int_C \nabla f \cdot d\mathbf{s} = f(\mathbf{c}(b)) - f(\mathbf{c}(a))$$

9. **Green's Theorem:** Let $D \subset \mathbb{R}^2$ with oriented boundary ∂D.

$$\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

10. **Stokes' Theorem:** Let $S \subset \mathbb{R}^3$ be an oriented surface with oriented boundary ∂S.

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{s} = \iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$$

11. **Divergence Theorem:** Let $W \subset \mathbb{R}^3$ be a region with boundary ∂W oriented away from W.

$$\iint_{\partial W} \mathbf{F} \cdot d\mathbf{S} = \iiint_W \text{div}\mathbf{F}\,dV$$

12. **Generalized Stokes' Theorem:** Let M be an oriented k-dimensional manifold with oriented boundary ∂M. If α is a $(k-1)$-form on M, then

$$\int_M d\alpha = \int_{\partial M} \alpha$$