Solutions

1.) $Q_{\mathcal{E}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q_{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

3.) Suppose $Q(v,w) = 0 \ \forall \ w \neq 0$.
Then $Q(v,v) = 0$. Since Q is definite, we have $v = 0$.

2.) Let $\mathcal{B} = \{b_1, \dots, b_n\}$ be a basis for $\mathbb{Z}^n$.
Suppose Q is positive definite.
The (i,i)-th entry of $Q_{\mathcal{B}}$ is $Q(b_i, b_i) > 0$

4.) Suppose Q is positive definite. Let $\mathcal{B}$ be a basis for $\mathbb{Z}^n$.
Let λ be an eigenvalue of $Q_{\mathcal{B}}$.
Let $v \neq 0$ be a corresponding eigenvector.
Then $0 < Q(v,v) = v^T Q_{\mathcal{B}} v = v^T(\lambda v) = \lambda(v \cdot v)$
Since $v \cdot v > 0$, we have $\lambda > 0$.