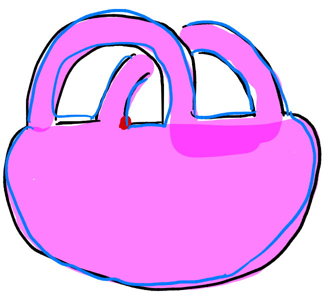
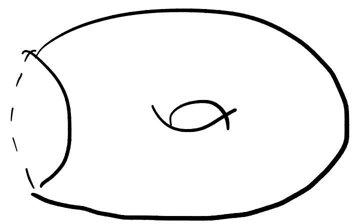
Problems:

1 (a) Identify the following surfaces up to homeomorphism (in terms of the surfaces described by the classification of surfaces.

(b) Compute their Euler characteristic.



$\underset{\text{homeo}}{\simeq}$

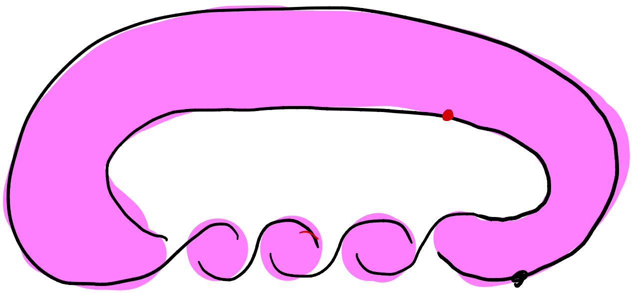


orientable

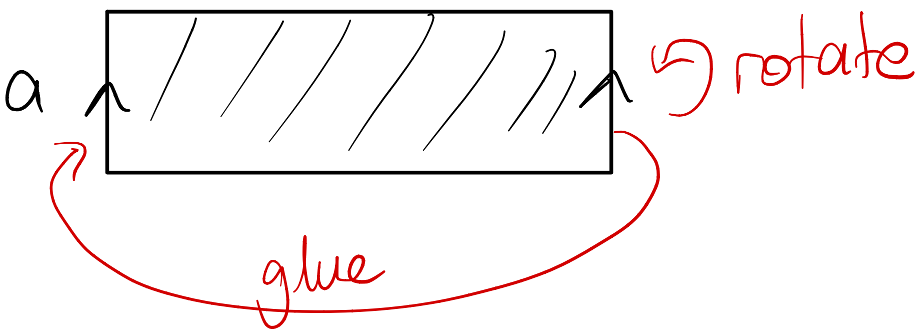
disks = 1

bands = 2

$$\chi = 1 - 2 = -1$$



orientable

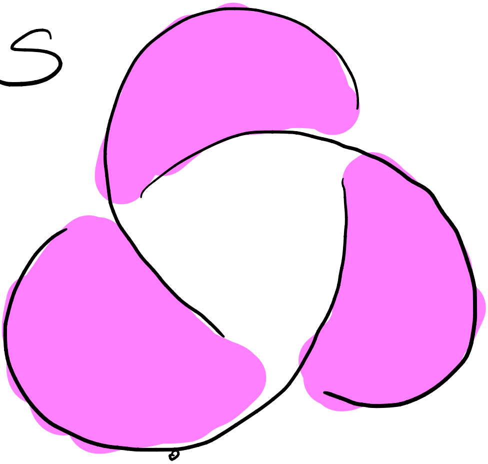


$v = 2$
$e = 3$
$f = 1$

$\#\text{disk} = 1$
$\#\text{bands} = 1$
$\chi = 1 - 1 = 0$

$$\chi(S) = 2 - 3 + 1 = 0$$

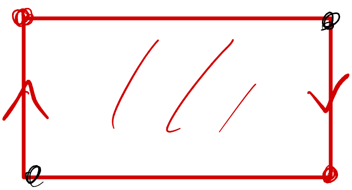
S



Möbius Band

Sifert Surface of trefoil knot

non-orientable



$\# v = 2$

$\# e = 3$

$\# f = 1$

$\chi(MB) = 0$

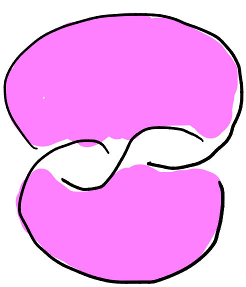
$\#$ disks $= 2$

$\#$ bands $= 3$

$\#$ bdry comp $= 1$

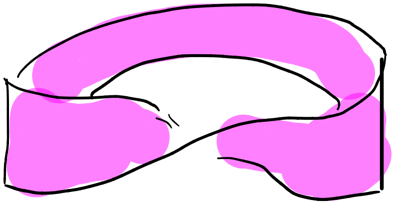
$\chi(S) = 2 - 3 + 1 = 0$

Remark:



another Seifert Surface of trefoil knot

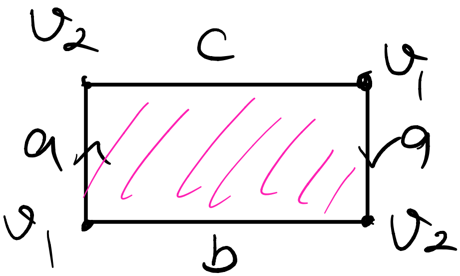
-orientable



Möbius Band

non-orientable

$\chi(MB) = -1$



$\# v = 2$

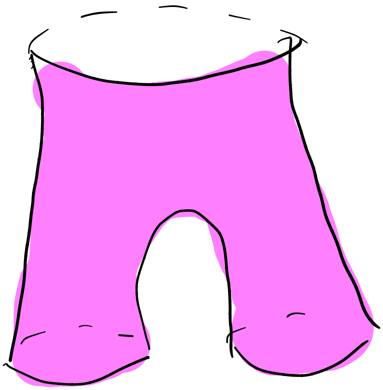
$\# e = 3$

$\# f = 1$

$\chi = 2 - 3 + 1$

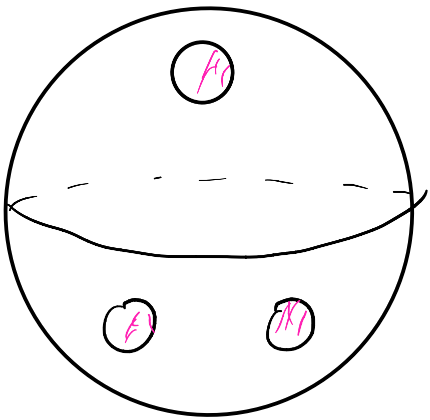
$\chi(MB) = 2 - 3 + 1 = -1$

S



$\chi(S) = -1$

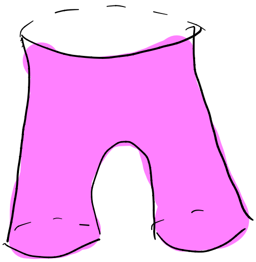
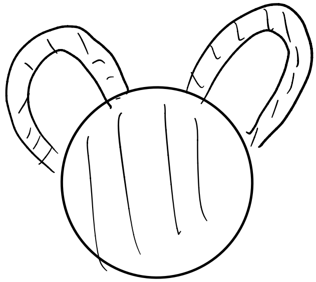
orientable



$S^2 - \bigcup_{i=1}^{3} D^2$

$$\chi(S) = \chi(S^2) - 3\chi(D^2)$$
$$= 2 - 3 = -1$$

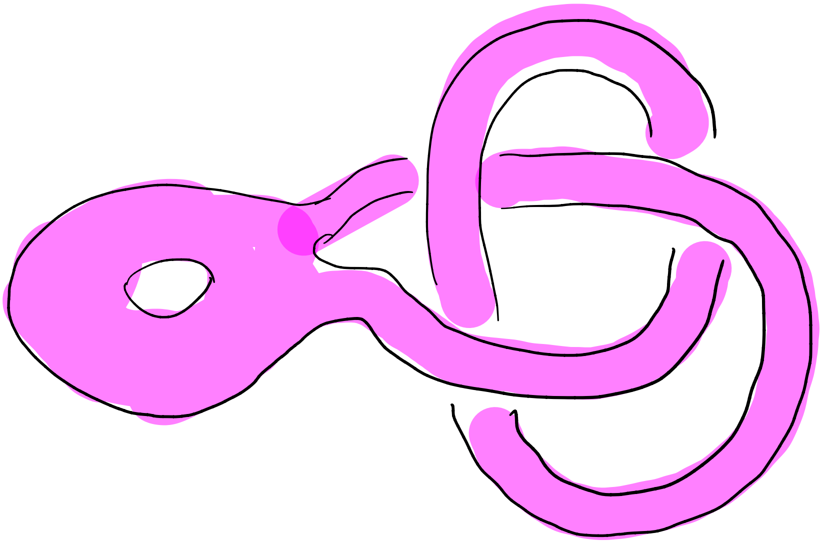
another solution:

 =  = 

#disks = 1

#bands = 2

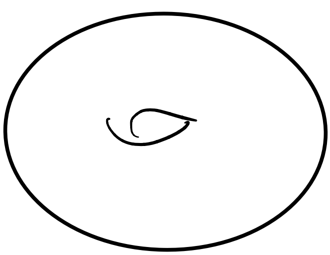
$$\chi(S) = 1 - 2 = -1$$



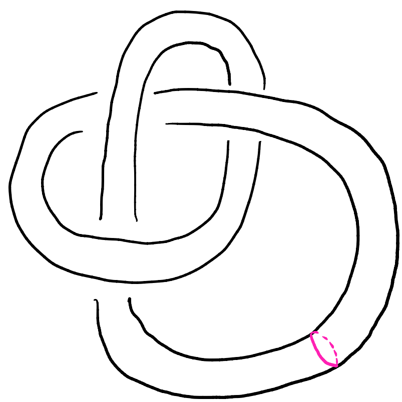
$\chi = -2$

orientable

T^2



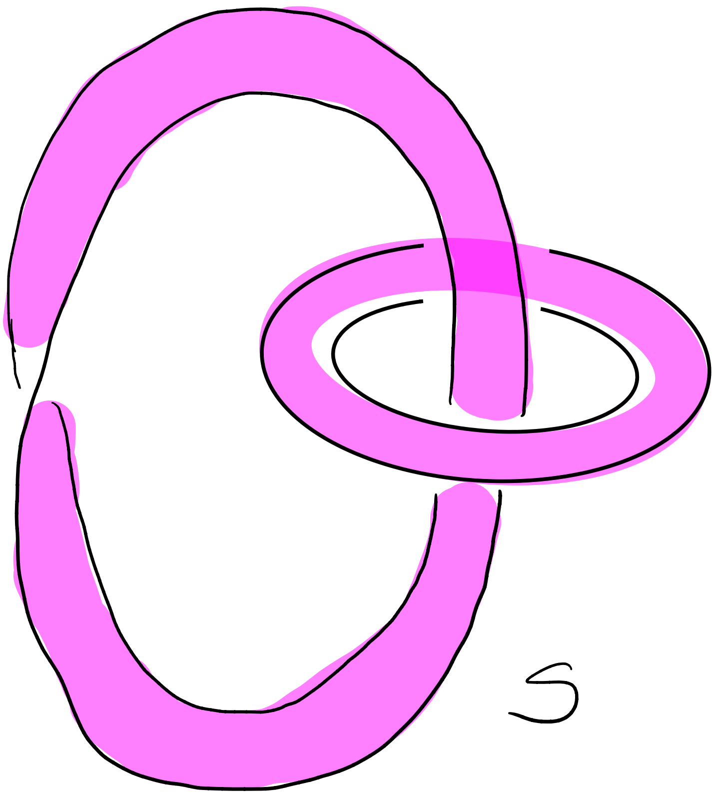
$\#$



S

$$\chi = \chi(T^2) + \chi(S) - 2\chi(D^2)$$

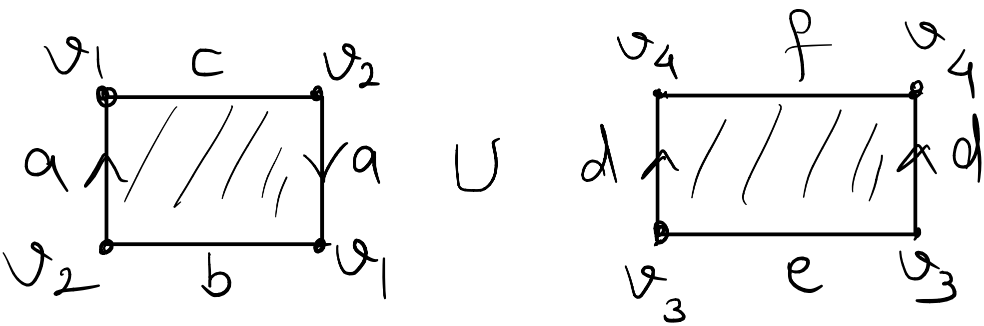
$$= 0 + 0 - 2 = -2$$



S

$\chi = 0$

NOT orientable



$$\chi(S) = 4 - 6 + 2 = 0$$

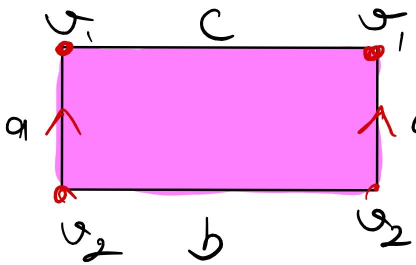
(2) Prove the above surfaces are OR are NOT orientable by showing they are 1-OR 2-sided.

Solution:

See Problem (1)

③ Calculate the Euler characteristic of the surfaces listed in the lecture notes.

ⓐ



Cylinder

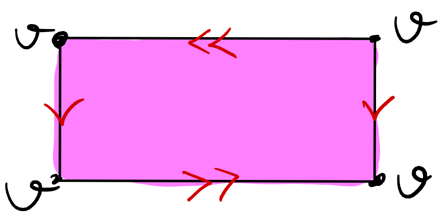
$\# v = 2$

$\# e = 3$

$\# f = 1$

$$\chi(C) = 2 - 3 + 1 = 0$$

ⓑ



KB $\chi = 0$

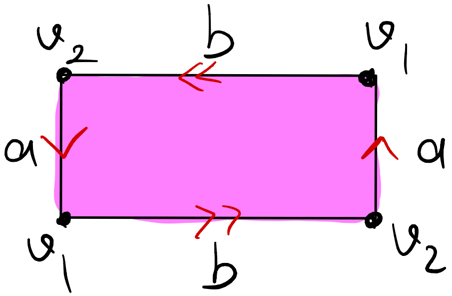
$\# v = 1$

$\# e = 2$

$\# f = 1$

$$\chi(KB) = 1 - 2 + 1 = 0$$

(c)



$\mathbb{RP}^2$

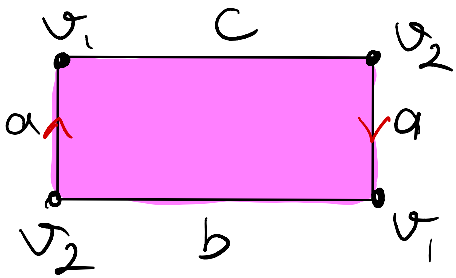
$\# v = 2$

$\# e = 2$

$\# f = 1$

$$\chi(\mathbb{RP}^2) = 2 - 2 + 1 = 1$$

(d)



MB

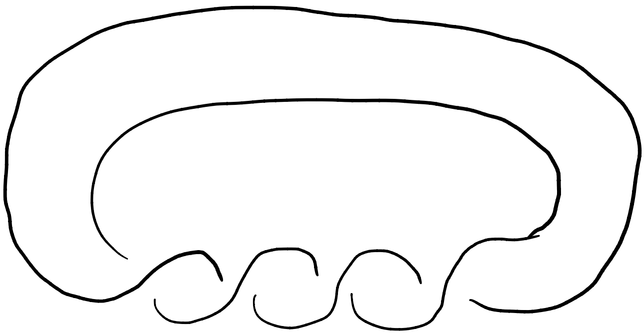
$\# v = 2$

$\# e = 3$

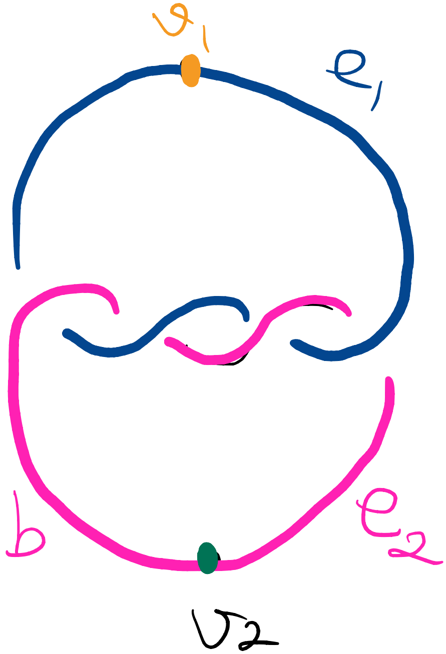
$\# f = 1$

$$\Longrightarrow \chi(MB) = 2 - 3 + 1 = 0$$

(e)



$\chi = 0$

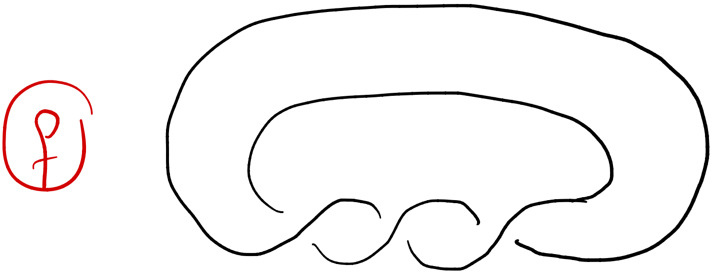


$\# v = 2$

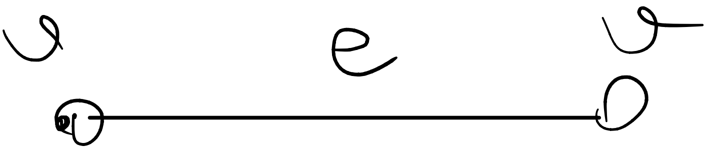
$\# e = 2$

$\# f = 0$

$$\chi = 2 - 2 + 0 = 0$$



$\chi = 0$



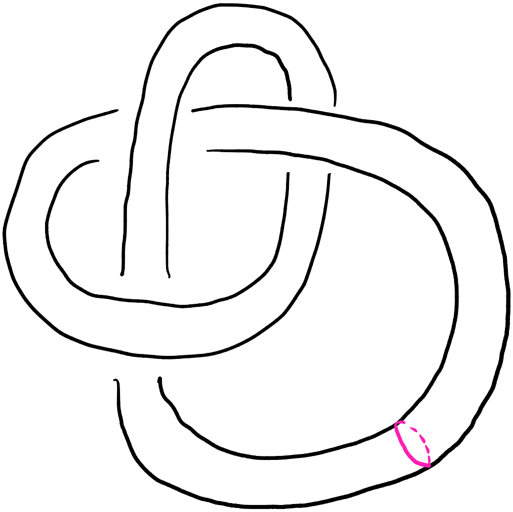
$\# v = 1$

$\# e = 1$

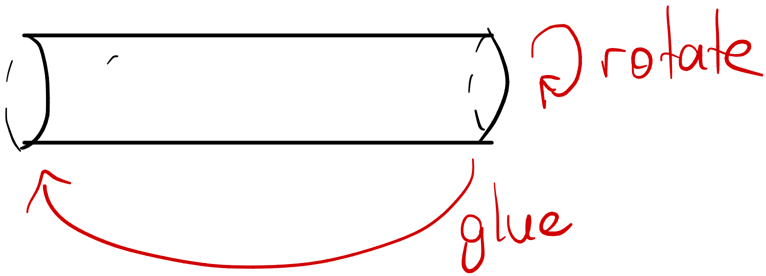
$\# f = 0$

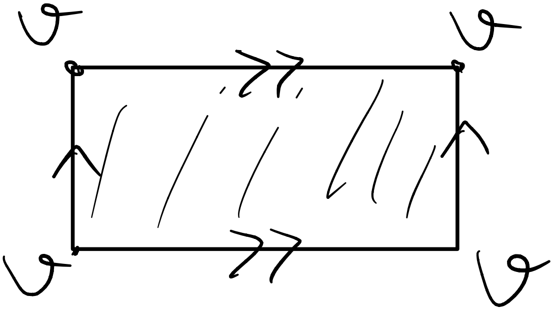
$\chi = 1 - 1 + 0 = 0$

9



$\chi = 0$





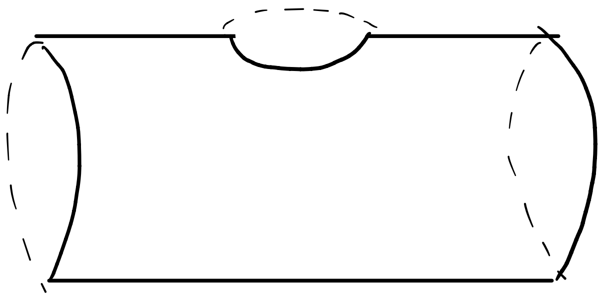
$\# v = 1$

$\# e = 2$

$\# f = 1$

$\chi = 1 - 2 + 1 = 0$

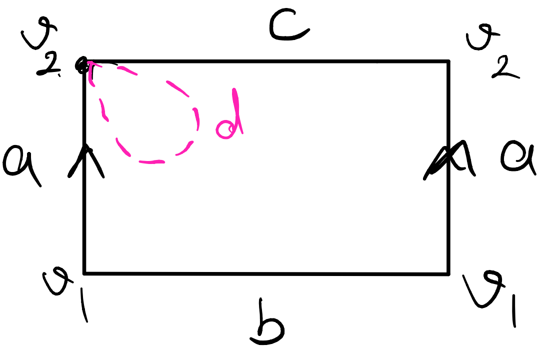
h



S

$\chi = -1$

$$S = C - D^2 = S^2 - \bigcup_{i=1}^{3} D^2$$

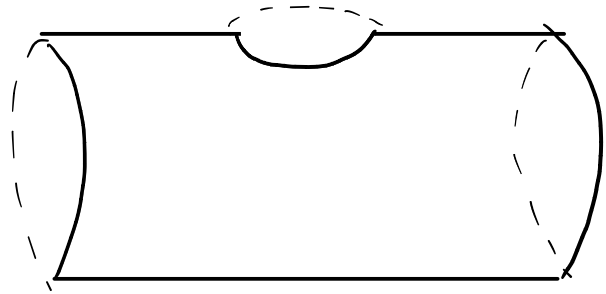


$\# v = 2$

$\# e = 4$

$\# f = 1$

$$\chi(S) = 2 - 4 + 1 = -1$$



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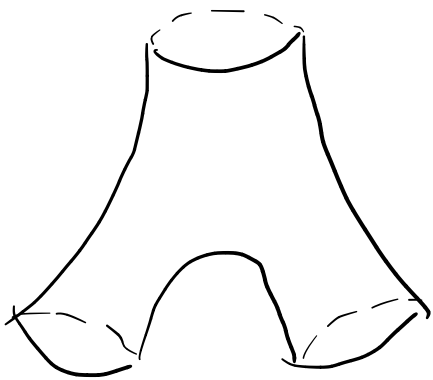

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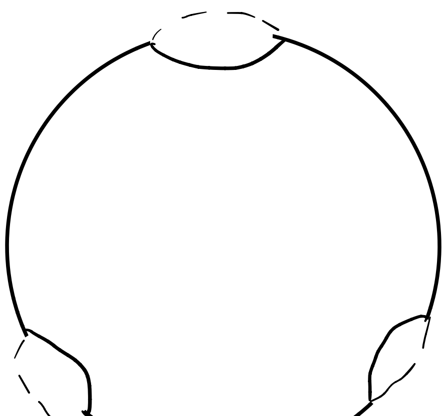

④ Compute the Euler Characteristic of

ⓐ $\#_n P = P \# P \# \cdots \# P$

ⓑ $\#_n P - \left(\bigcup_{i=1}^{m} D_i \right)$

ⓒ $\#_n T^2 - \left(\bigcup_{i=1}^{m} D_i \right)$

ⓓ $S^2 - \left(\bigcup_{i=1}^{m} D_i \right)$

Solution:

ⓐ $\chi(\#_n P) = n\chi(P) - (n-1)\chi(D^2)$

$= n \cdot 1 - (n-1)\, 2 = 2 - n$

ⓑ $\chi\left(\#_n P - \bigcup_{i=1}^{m} D_i\right) =$

$= \chi(\#_n P) - m\,\chi(D^2) = 2 - n - m$

(c) $\chi\left(\#_n T^2 - \left(\bigcup_{i=1}^{m} D_i\right)\right) =$

$= \chi\left(\#_n T^2\right) - m\,\chi(D^2)$

$= n\,\chi(T^2) - 2(n-1)\,\chi(D^2) - m\,\chi(D^2)$

$= n \cdot 0 - 2(n-1) - m$

$= 2 - 2n - m$

(d) $\chi\left(S^2 - \left(\bigcup_{i=1}^{m} D_i\right)\right)$

$= \chi(S^2) - m\,\chi(D_i)$

$= 2 - m$