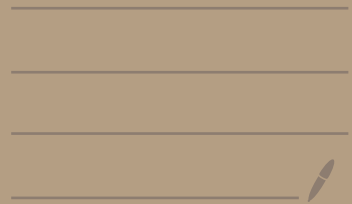


HW Solutions

Knots-Links - Spanning Surfaces



Problems

① Find a Seifert Surface for the following Knots/Links.

What is the genus of the link/knot?

②

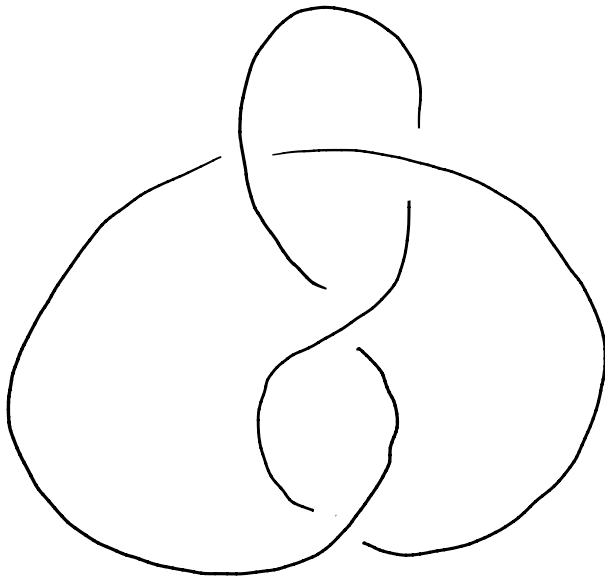
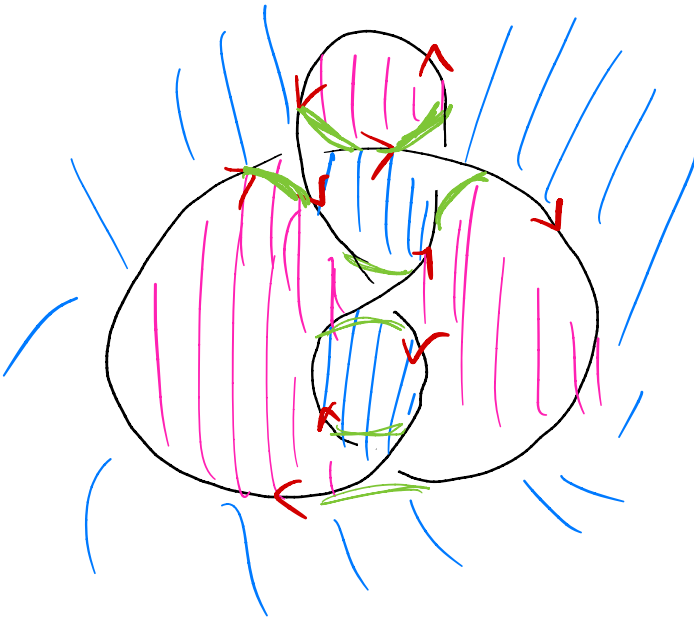
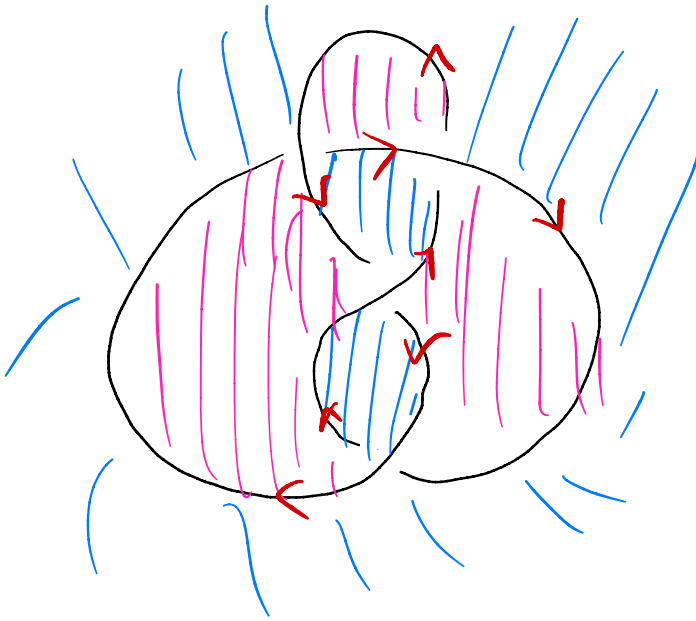
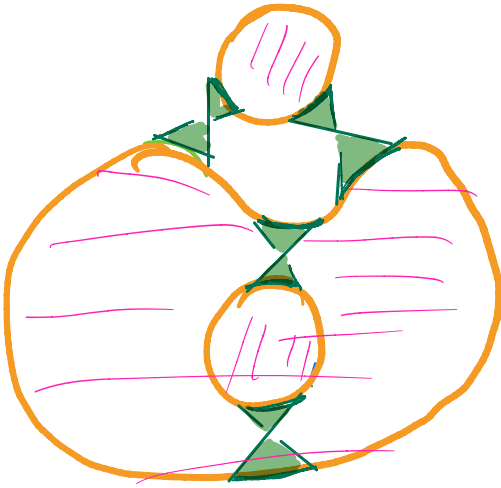


Figure 8

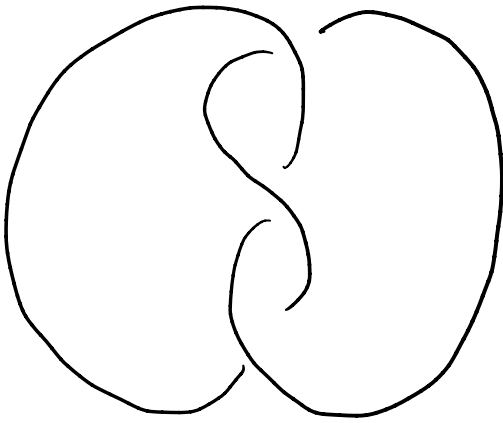




3 disks

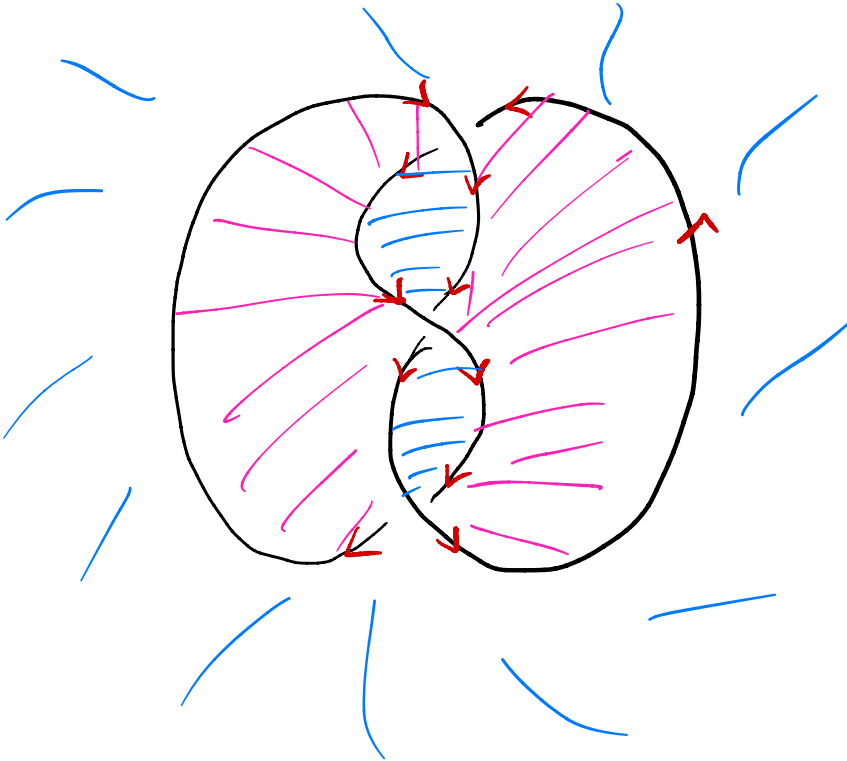
4 bands

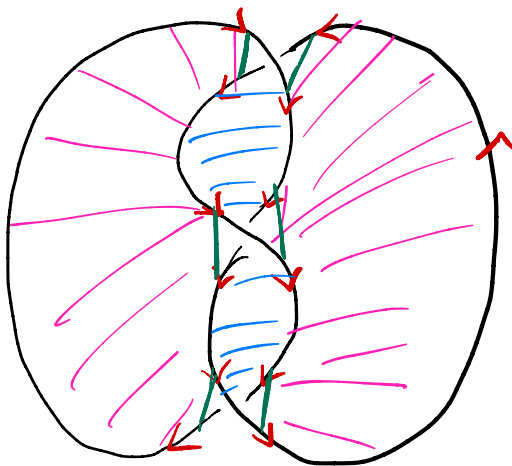
b



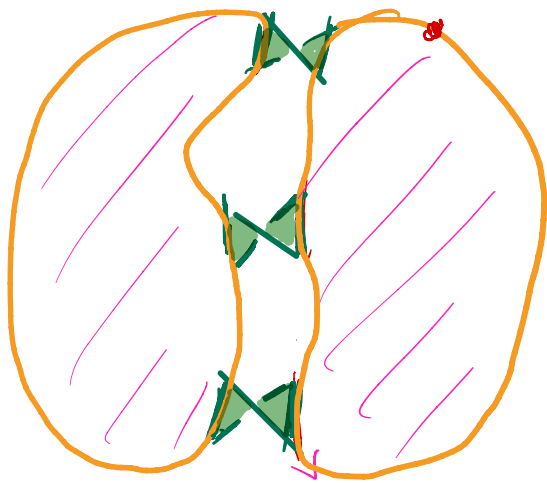
Trefoil
Knot

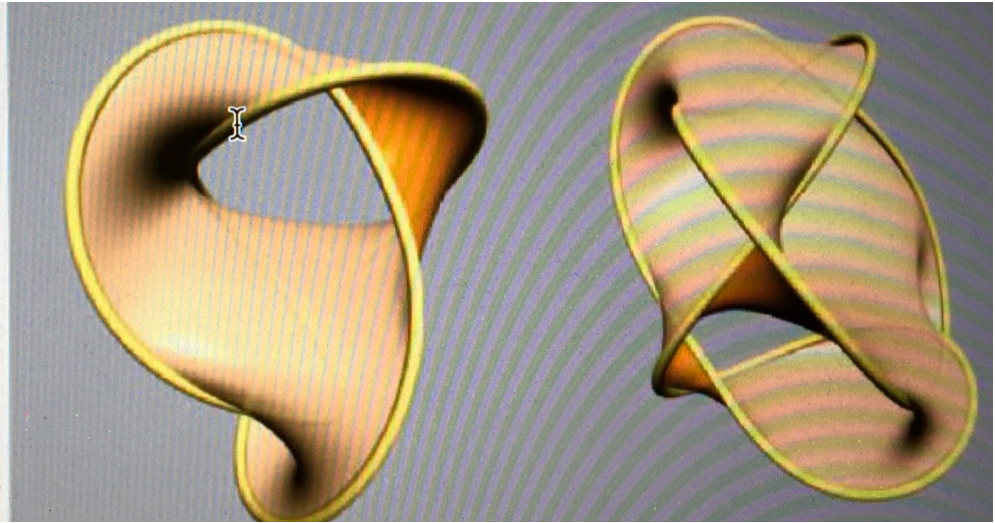
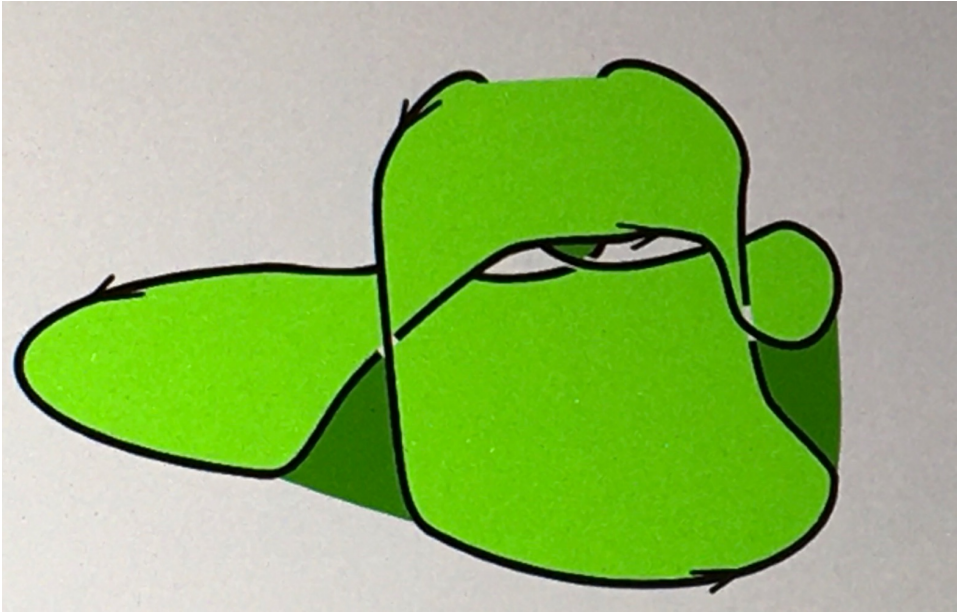
(Compare with the one we draw
in the lecture)



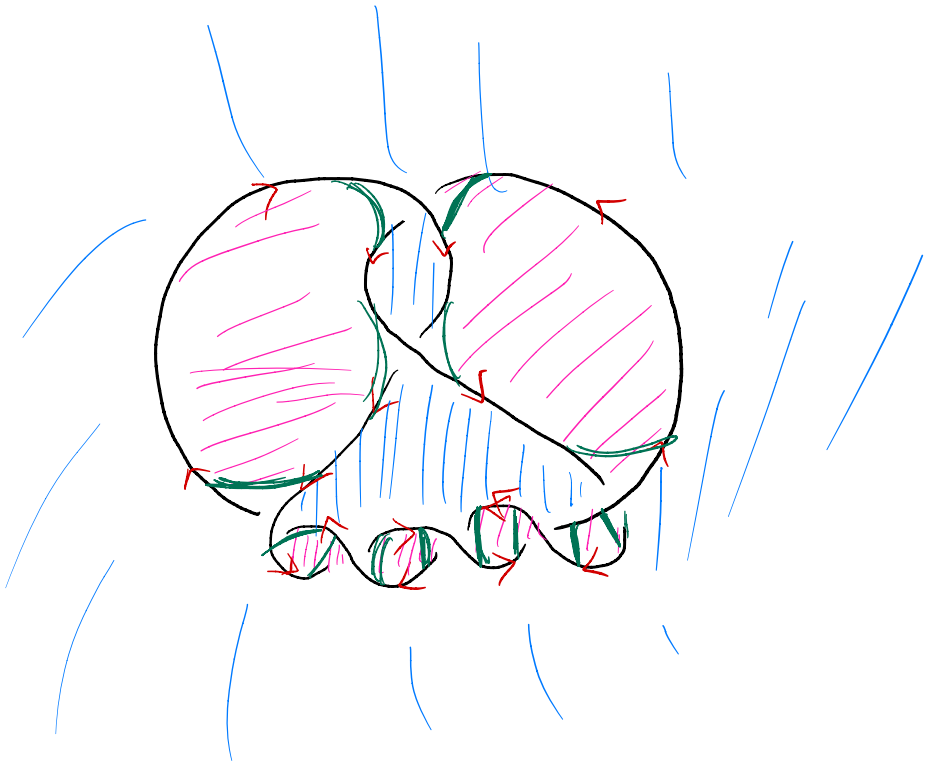
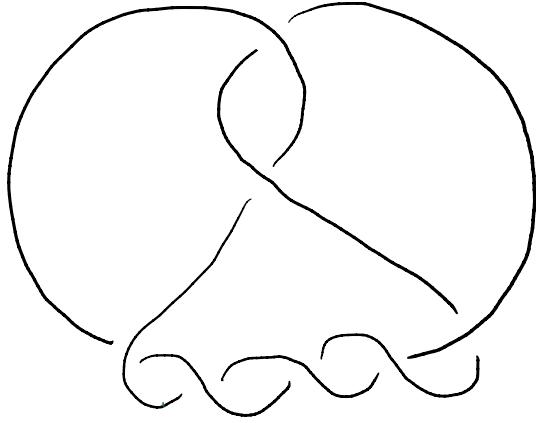


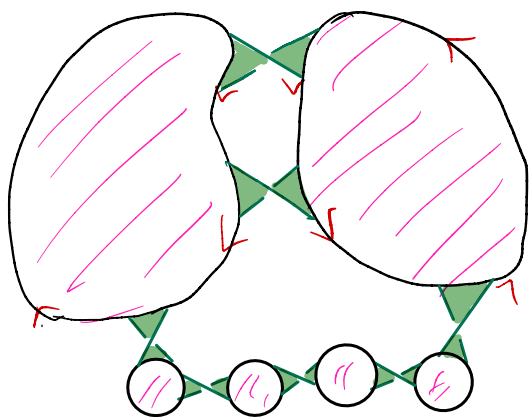
2-disks
3-bands





(c)

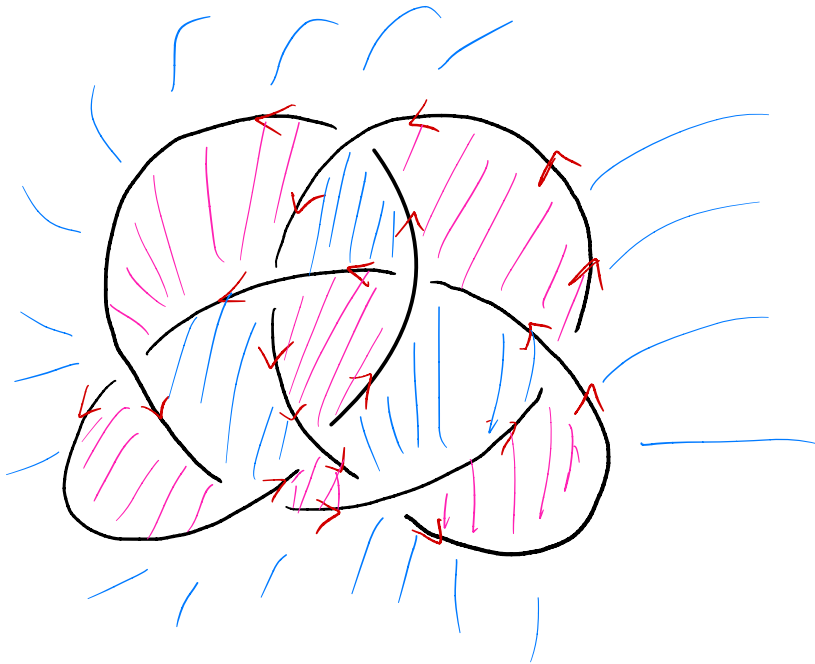
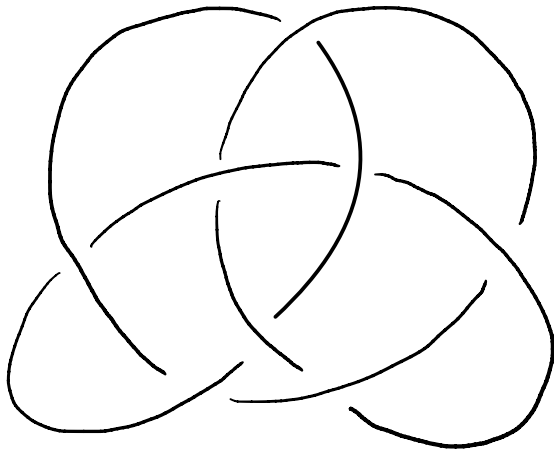


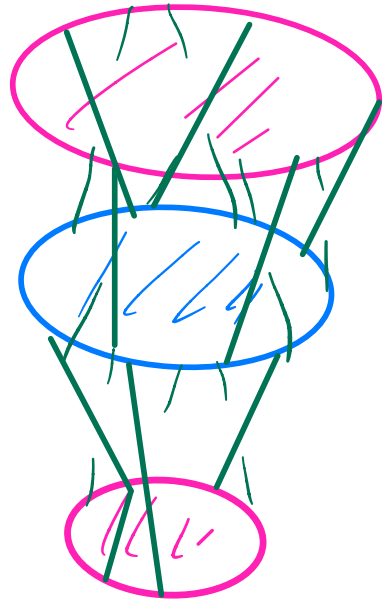
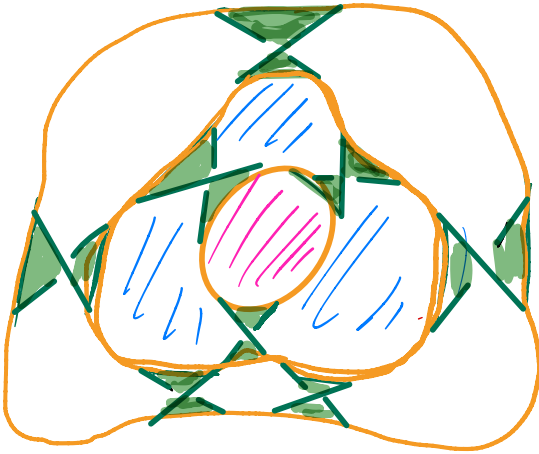
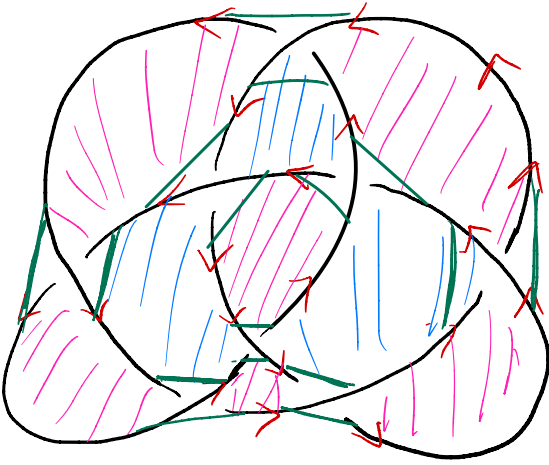


6 disks

7 bands

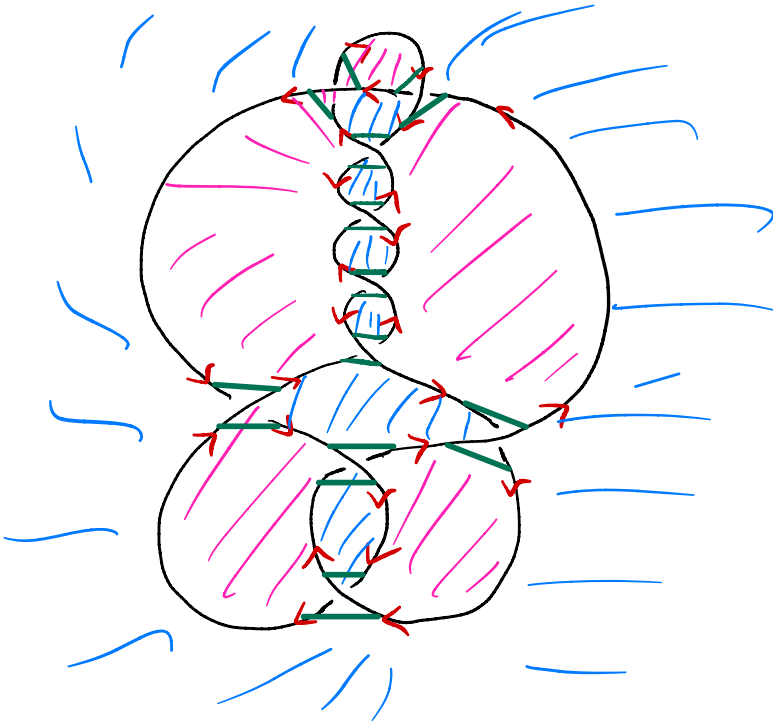
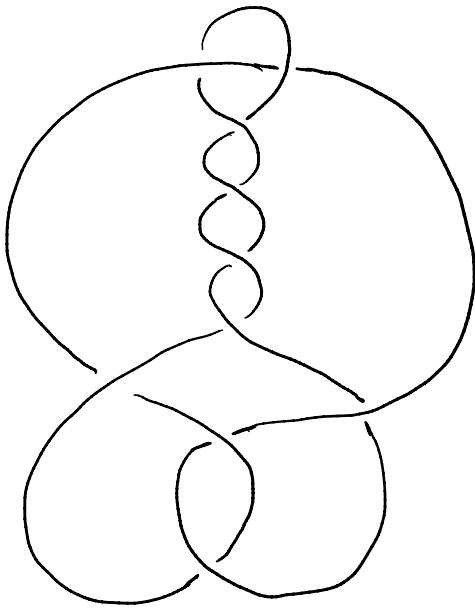
d

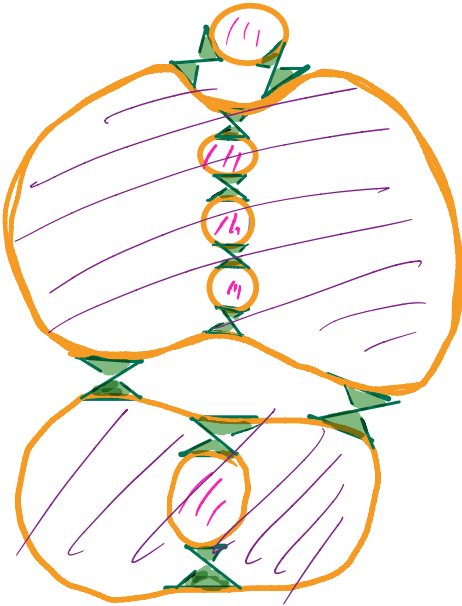




3 disks
8 bands

e



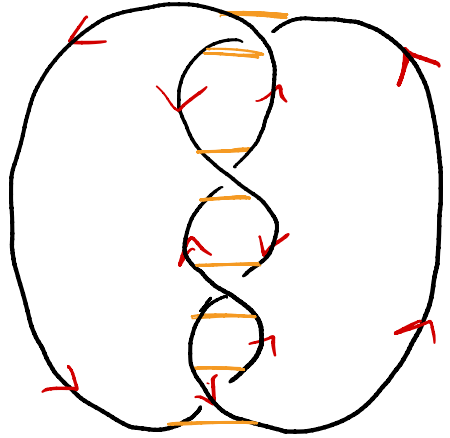
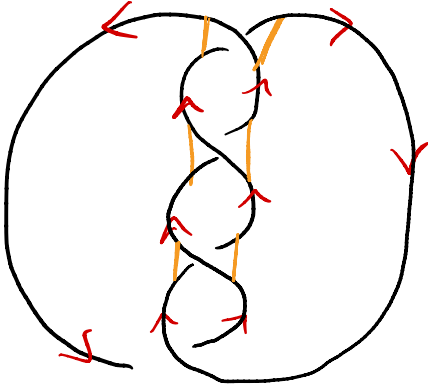


7 - disks
1 - band

② For each knot in Problem 1, draw a non-orientable spanning surface.

Solution:

(a)



$$\# \text{ disks} = 2$$

$$\# \text{ bands} = 4$$

$$\# \text{ bdry comp.} = 0$$

$$\chi = 2 - 4 + 0 = -2$$

$$\Rightarrow g = 2$$

$$\# \text{ disk} = 4$$

$$\# \text{ bands} = 4$$

$$\# \text{ bdry comp} = 0$$

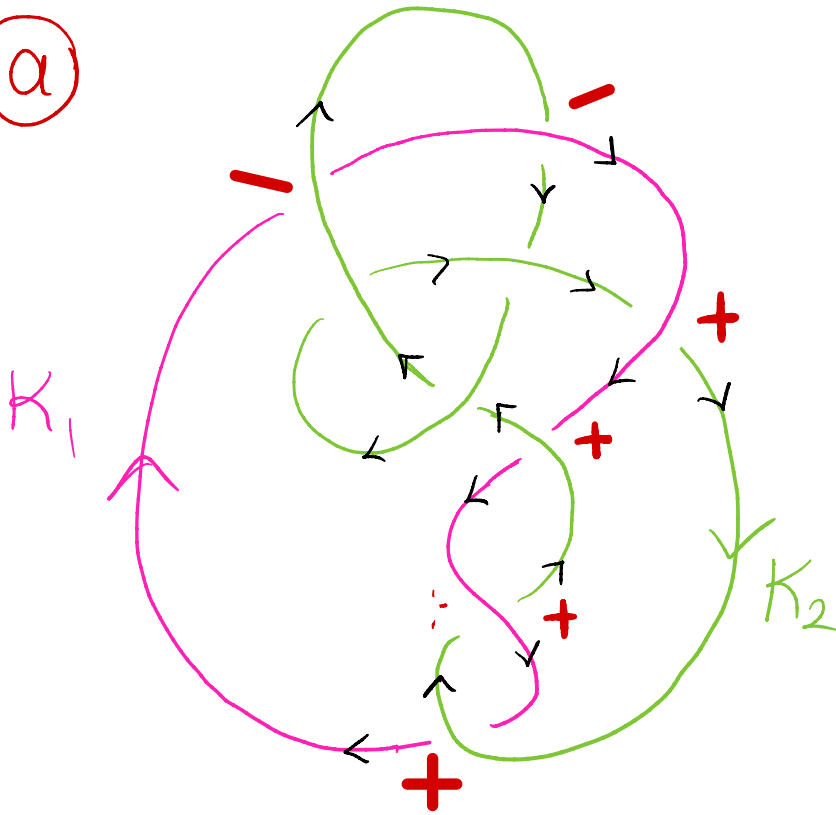
$$\chi = 4 - 4 - 0 = 0$$

$$\Rightarrow g = 2$$

\Rightarrow This link has $g=0$.

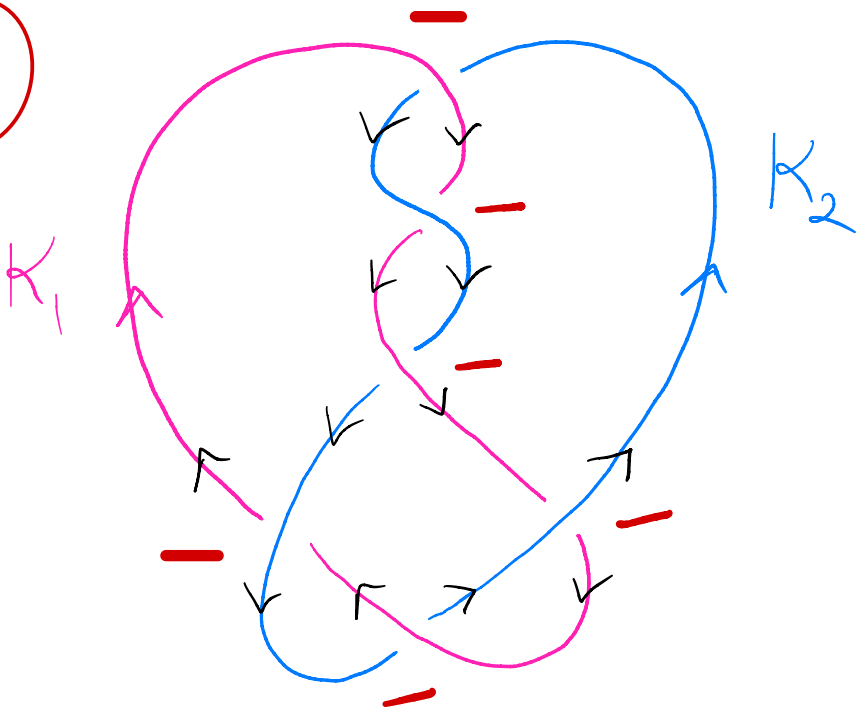
③ Calculate the linking number of the following links.

①

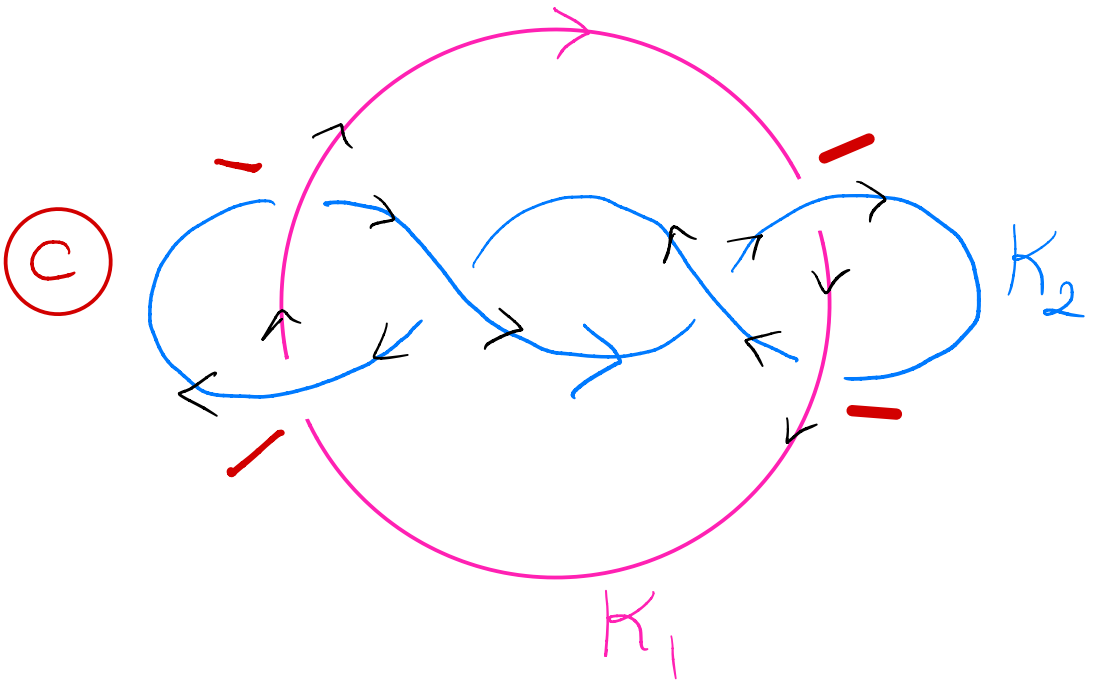


$$lk(K_1, K_2) = \frac{4 + 2}{2} = 3$$

(b)

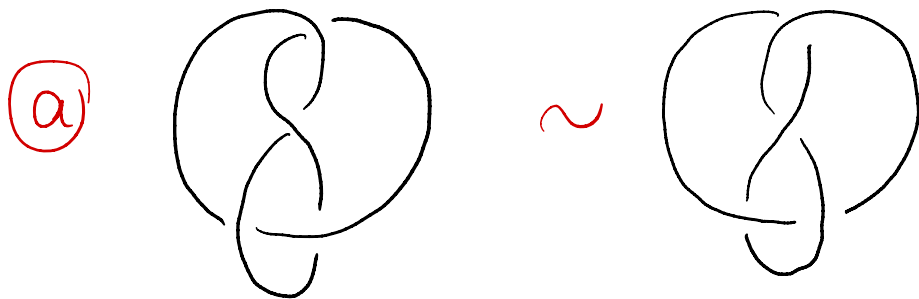


$$lk(K_1, K_2) = \frac{0 - 6}{2} = -3$$

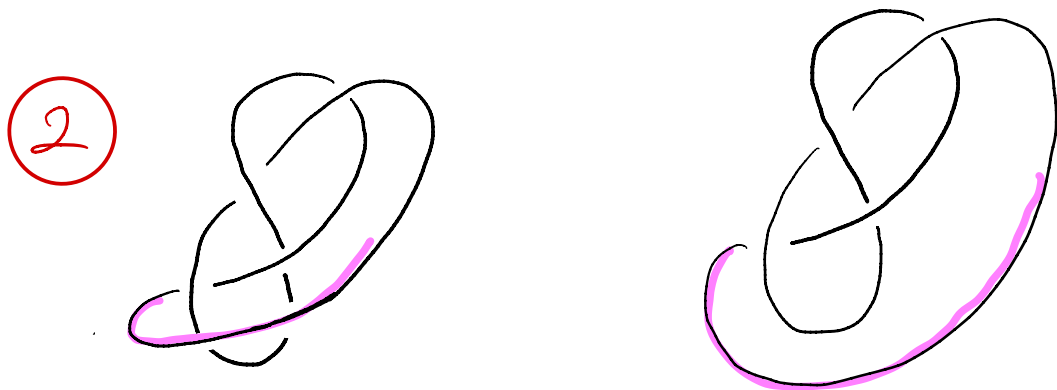
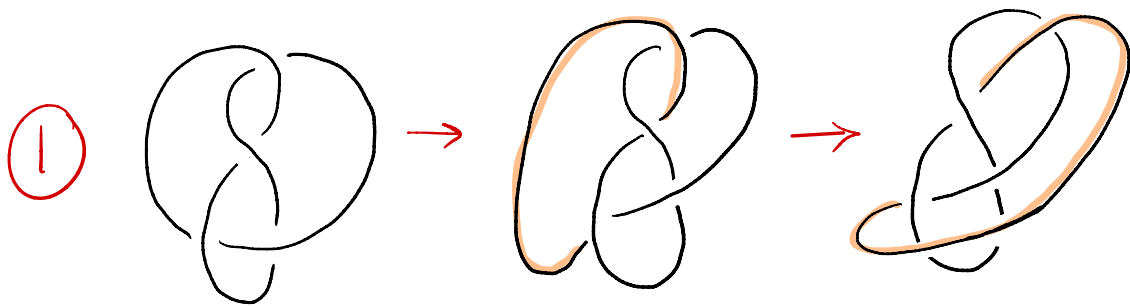


$$lk(K_1, K_2) = \frac{0 - 4}{2} = -2$$

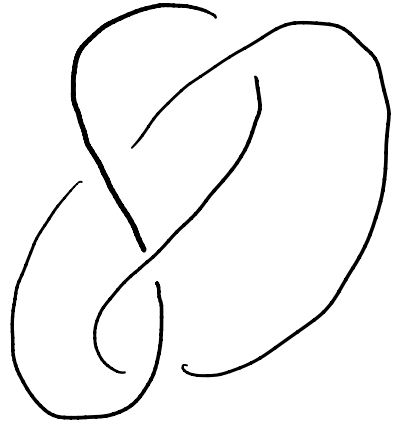
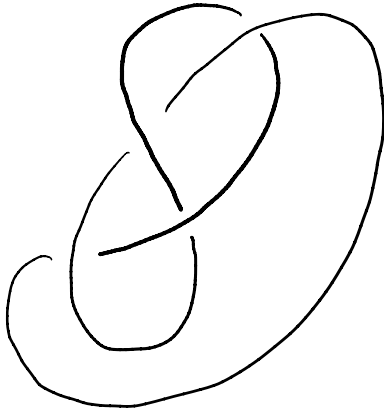
④ Show that the following knots are same using isotopies and Reidemeister moves.



Solution:



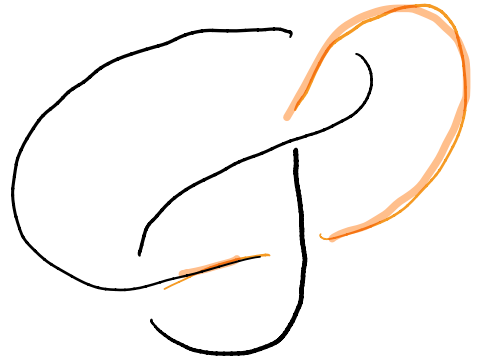
③



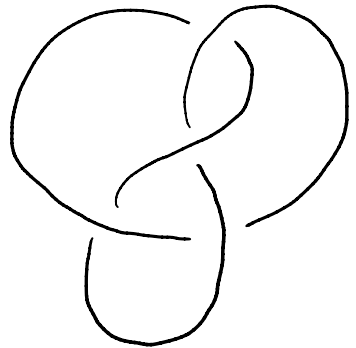
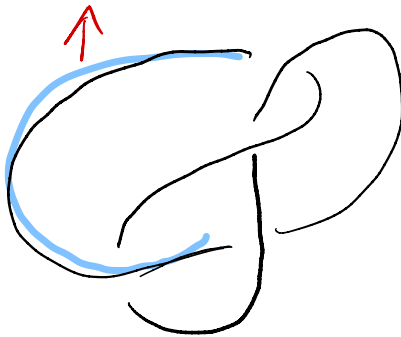
④



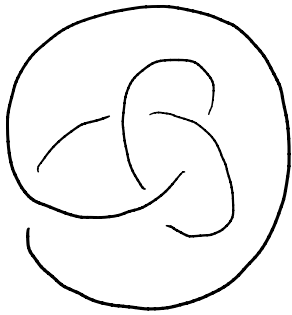
→
rotate



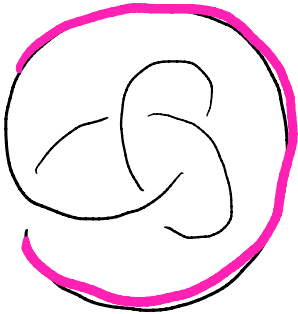
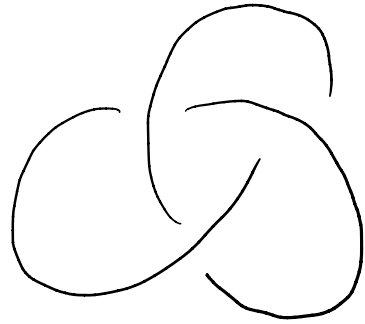
⑤



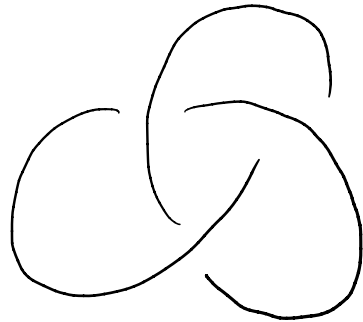
b



~

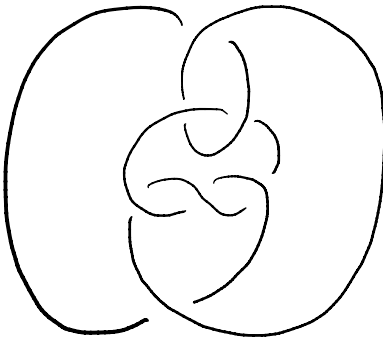


→

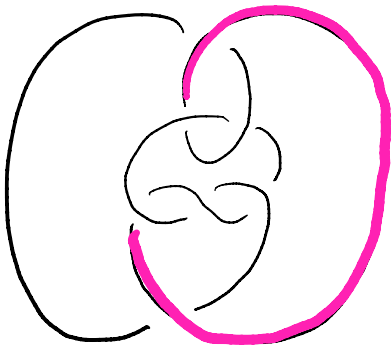
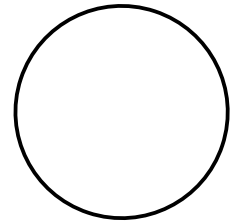


pull it

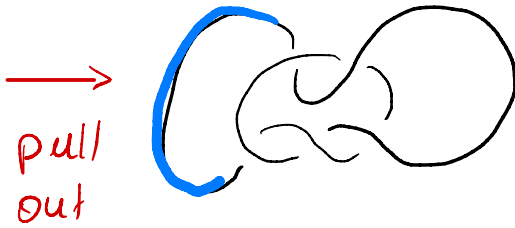
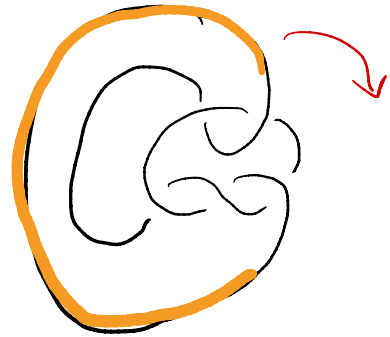
③



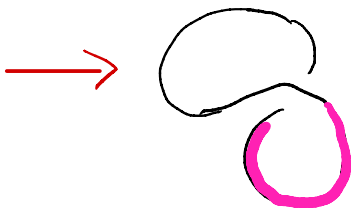
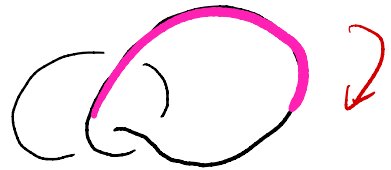
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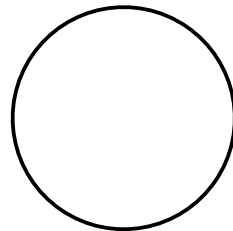
→
pull
inside



→



→
flip



⑤ Download KLO (Knot-Like-Objects)

KLO-software.net

and play around with it.

⑥ Check

Knotinfo.math.indiana.edu

At the top bar

there is an option for links.

