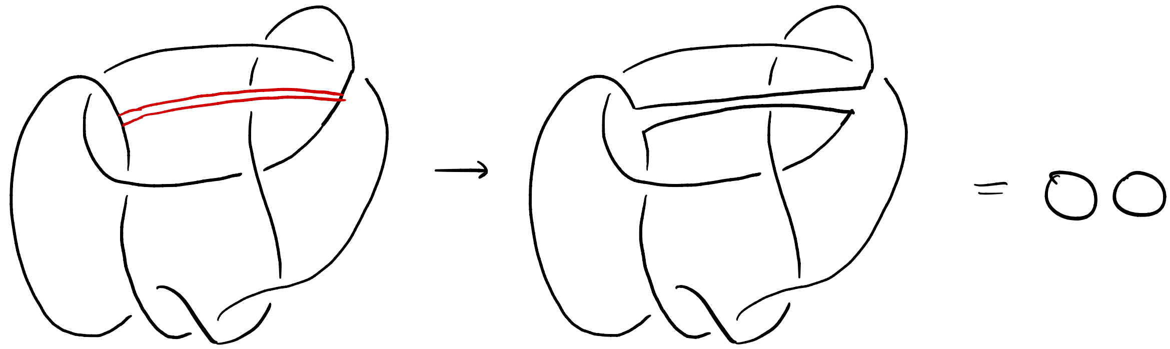
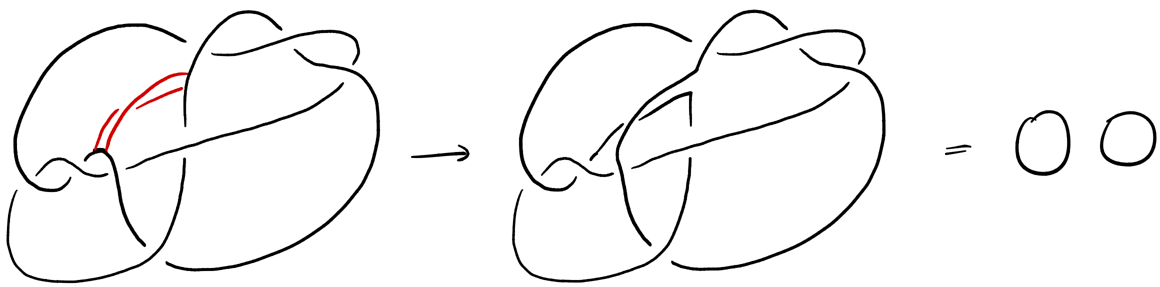
Solutions

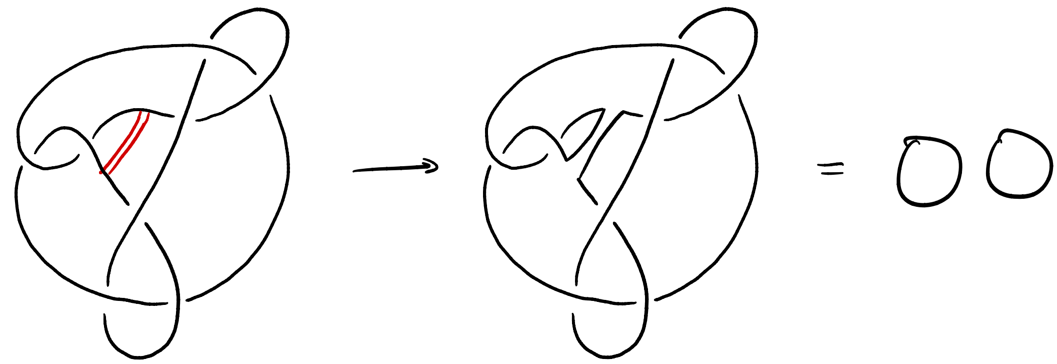
1.) $8_8 =$



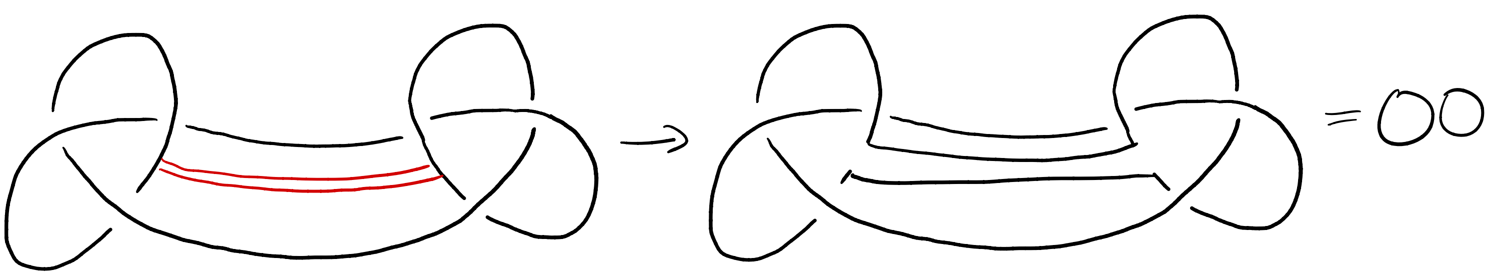
$8_9 =$



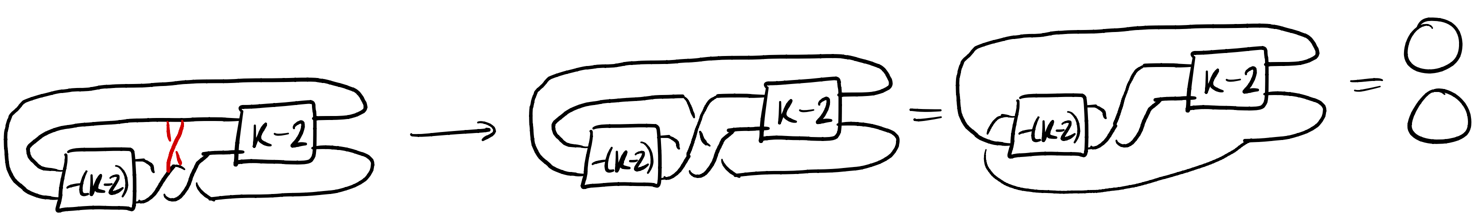
$8_{20} =$



2.)



3.)



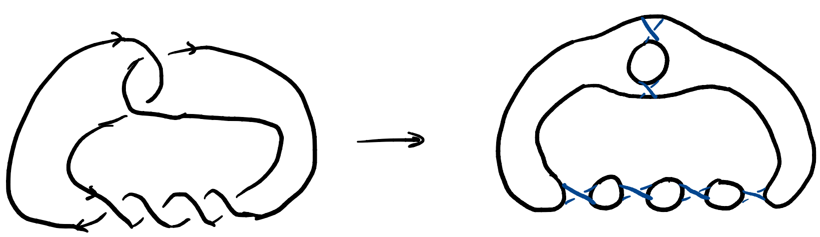
L_K is a knot if K is even and a two component link if K is odd. Hence L_K bounds a disk if K is even and the disjoint union of a disk and Mobius band if K is odd.

4.) a) Let Σ be a spanning surface for K in S^3 such that $g(\Sigma) = g_3(K)$.
Pushing Σ into B^4 gives a surface in B^4.
Hence by definition, $g_4(K) \leq g(\Sigma) = g_3(K)$.

b) Take the unknot U. Since U bounds a disk in S^3 and in B^4, $g_3(K) = 0 = g_4(K)$

c) Consider 6_1 = 

We know 6_1 is slice (see notes) $\Rightarrow g_4(K) = 0$.
Clearly $6_1 \neq$ unknot $\Longrightarrow g_3(K) \geq 1$.
Using Seifert's algorithm, we can find an orientable surface F in S^3 bounded by K:



We can calculate $\chi(F) = 5 - 6 = -1$
Since F is orientable and has one boundary component, we have $g(F) = 1$.
Hence $g_3(K) = 1$.