HW Solutions

Handlebody Decomposition

Problems

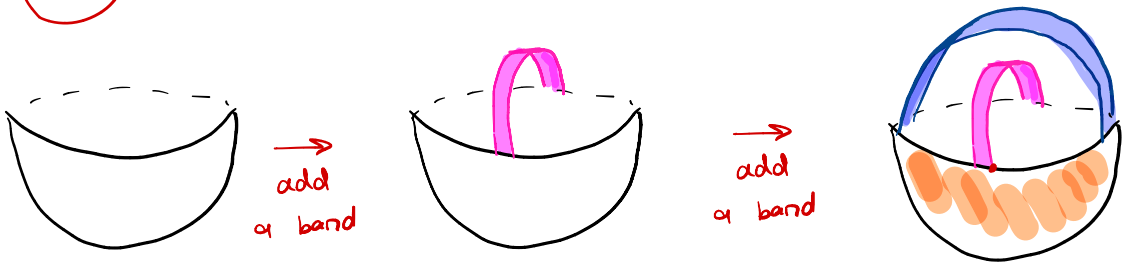
1. Draw a handlebody decomposition of

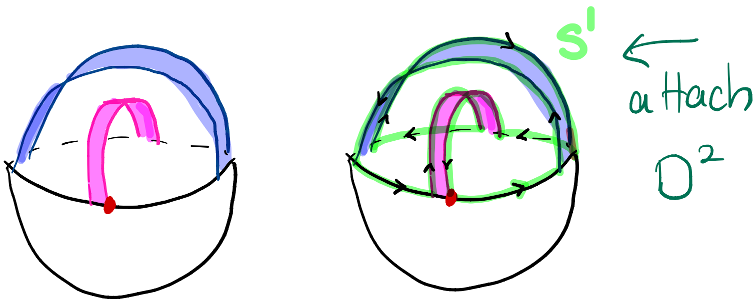
   a. T^2

   b. Σ_2

   c. $\mathbb{RP}^2$

1a





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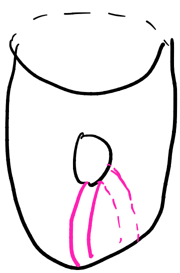
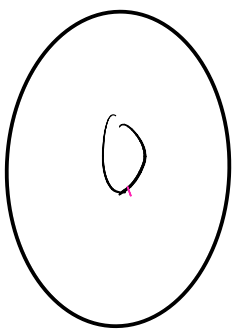
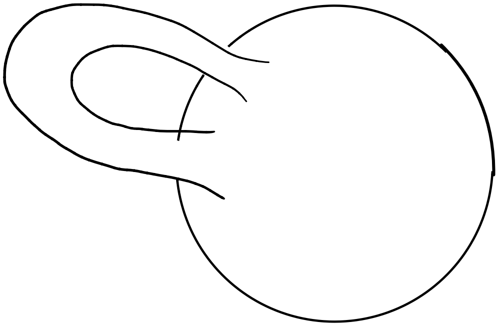


cap it off with D^2

$\Rightarrow$  T^2

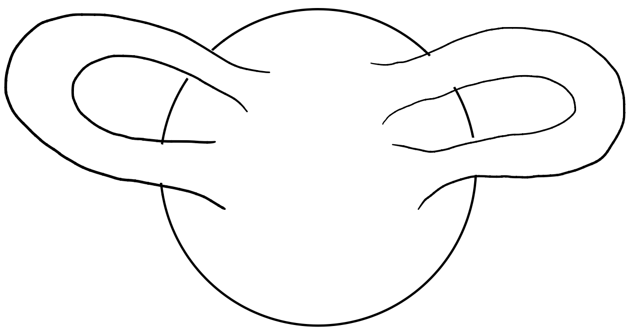
1a



1b

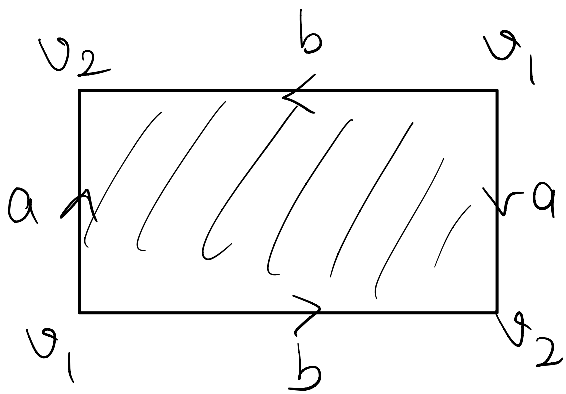
$\cup$ 2 disks

OR



1c $\mathbb{RP}^2$

$\cup \; D^2$



2. Determine the number of each k-handle for

a. $\mathbb{RP}^n$

b. $\mathbb{CP}^n$

$$\mathbb{CP}^n = \mathbb{C}^{n+1} - \{0\} \Big/ z \sim w$$

$z \sim w \iff w = \lambda z$ for some $\lambda \in \mathbb{C}^{n+1} - \{0\}$

i.e.

$z \sim w \iff$ they are complex linearly DEP.

Q: What is $\mathbb{CP}^1$?

a) $\mathbb{R}P^n$

$$\mathbb{R}P^n = (1\text{-}0h) \cup (1\text{-}1h) \cup \cdots \cup (1\text{-}nh)$$

there are one k-handle

for each $1 \le k \le n$

b) $\mathbb{C}P^n$

$$\mathbb{C}P^n = (1\text{-}0h) \cup (1\text{-}2h) \cup \cdots \cup (1\text{-}2n\,h)$$

Not $\dim(\mathbb{C}P^n) = 2n$

$$\mathbb{C}P^n = \mathbb{C}^{n+1}/\sim \; = \mathbb{R}^{2n+2}/\sim$$

There is one k handle

for each $0 \le k \le 2n$ if $k = \text{even}$

There is NO k handle

if $k = $ odd.

$$\mathbb{C}P^1 = \mathbb{C}^2 - \{0\} / x \sim \lambda x = S^3/S^1 = S^2$$

$= $ lines in $\mathbb{C}^2$

Each line in $\mathbb{C}^{n+1}$ intersects S^{2n+1}

$\mathbb{C}^2$ S^3

in a circle.

Hopf fibration:

$$\begin{tikzcd} S^1 \arrow[r] & S^3 \arrow[d] \\ & S^3/S^1 = \mathbb{C}P^1 \end{tikzcd}$$